Monomial and binomial ideals
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Exercises 1

Ideals and Varieties

Aufgabe 1.1

a) Install Macaulay2 on your computer.

   Hints:

   - I heard stories that this can be very painful on Windows machines. It may be feasible to run a virtual machine with Ubuntu just for Macaulay2.

   - Install Emacs as well and run Macaulay2 in Emacs. Google for "Macaulay2 emacs"

b) Run the tutorial "Ideals, Varieties, and Macaulay2" by Bernd Sturmfels (google for "Ideals, Varieties, and Macaulay2" to find a pdf).

Exercise 1.2
Find a counterexample to Hilbert's Nullstellensatz in the case of a non-algebraically closed field.

Exercise 1.3
Show that the implicit polar equation $r = \sin(2\theta)$ defines an affine variety $V \subset \mathbb{R}^n$. You could proceed as follows

a) Conjecture an implicit representation using polynomials in x, y coordinates of the plane.

b) The containment of the object defined by the polar equation in the conjectured affine variety should be easy, but you have to be careful with the other direction (why?).

What is the dimension of this object? Is it singular? Is it reducible into components? Use some software to plot the object (unfortunately Macaulay2 is not good at plotting, use Sage, Maple, Mathematica, . . .).